

INTRODUCTION TO GEOMETRY

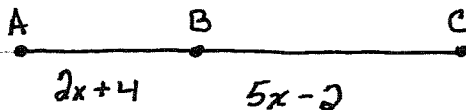
SOME KEY CONCEPTS TO REMEMBER

SEGMENT ADDITION

$$\text{IF } \overline{AC} = 30$$

$$\overline{AB} = 2x + 4$$

$$\overline{BC} = 5x - 2$$



THEN AN EQUATION CAN BE WRITTEN WHERE

$$\overline{AB} + \overline{BC} = \overline{AC}$$

$\overline{AB} = 2x + 4$	$\overline{BC} = 5x - 2$
$= 2(4) + 4$	$= 5(4) - 2$
$= 12$	$= 18$

$$2x + 4 + 5x - 2 = 30$$

$$7x + 2 = 30$$

$$7x = 30 - 2$$

$$7x = 28$$

$$x = 4$$

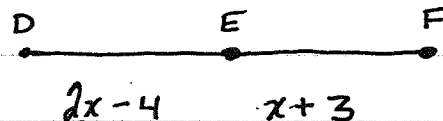
* THERE IS MORE TO THESE PROBLEMS THAN SOLVING AN EQUATION.
THE OBJECTIVE IS TO FIND THE LENGTH OF EACH SEGMENT.

MIDPOINT TO SEGMENT

IF A MIDPOINT IS GIVEN THEN IT WILL DIVIDE THE
SEGMENT INTO TWO EQUAL PARTS.

IF DF HAS A MIDPOINT AT E

$$\text{AND } \overline{DE} = 2x - 4, \overline{EF} = x + 3$$



THEN THE SEGMENTS CAN BE SET EQUAL TO ONE ANOTHER

$\overline{DE} = 2x - 4$	$\overline{EF} = x + 3$
$= 2(7) - 4$	$= 7 + 3$
$= 10$	$= 10$

$$\overline{DE} = \overline{EF}$$

$$2x - 4 = x + 3$$

$$2x - x = 3 + 4$$

$$x = 7$$

MIDPOINT FORMULA

* SOMETIMES THE MIDPOINT WILL BE DISCUSSED AS A POINT ON THE COORDINATE PLANE.

EXAMPLE: FIND THE MIDPOINT OF \overline{AC} IF $A(-3, 7)$ AND $C(4, -3)$ ARE THE ENDPONTS

SOLUTION $A(-3, 7)$ } ADD THE X-COORDINATE VALUES AND
 $C(4, -3)$ } DIVIDE BY 2, THEN DO THE SAME
FOR THE Y-COORDINATES.

$$x: \frac{-3+4}{2} = \frac{1}{2}$$

$$y: \frac{7+(-3)}{2} = \frac{4}{2} = 2$$

$$\text{MIDPOINT TO } \overline{AC} = \left(\frac{1}{2}, 2\right)$$

EXAMPLE 2: FIND THE MIDPOINT E TO \overline{DF} IF $D(11, 3)$ AND $F(5, -7)$ ARE ENDPONTS

SOLUTION $D(11, 3)$
 $F(5, -7)$

$$x: \frac{11+5}{2} = \frac{16}{2} = 8$$

$$y: \frac{3+(-7)}{2} = \frac{-4}{2} = -2$$

SO MIDPOINT TO \overline{DF} CALLED
 $E(8, -2)$

A VARIATION TO THIS PROBLEM OF MIDPOINTS IS WHEN ONE NEEDS TO FIND AN ENDPOINT GIVEN THE MIDPOINT AND ONE OF THE OTHER ENDPONTS.

EXAMPLE: FIND X IF THE MIDPOINT IS Y(5, -3) AND Z THE OTHER ENDPOINT TO \overline{XZ} IS Z(-2, -5).

SOLUTION X(x, y)
Z(-2, -5) MIDPOINT Y(5, -3)

$$\text{SO } \frac{x+(-2)}{2} = 5 \quad \text{AND} \quad \frac{y+(-5)}{2} = -3$$

$$\frac{x-2}{2} = 5$$

$$x-2=10$$

$$x=12$$

$$\frac{y-5}{2} = -3$$

$$y-5=-6$$

$$y=-1$$

$$\boxed{X(12, -1)}$$

EXAMPLE 2: MIDPOINT TO \overline{AC} IS FOUND AT B(1, -5)
ENDPOINT A(3, 2) FIND COORDINATES FOR C.

SOLUTION A(3, 2) MIDPOINT (1, -5)
C(x, y)

$$\text{SO } \frac{x+3}{2} = 1 \quad \text{AND} \quad \frac{y+2}{2} = -5$$

$$x+3=2$$

$$x=2-3$$

$$x=-1$$

$$y+2=-10$$

$$y=-10-2$$

$$y=-12$$

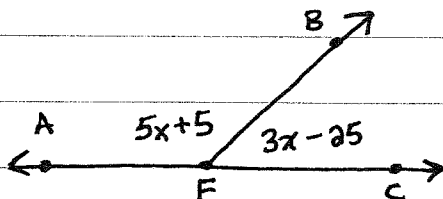
$$\boxed{C(-1, -12)}$$

ANGLES - THE ARCH MEASURE BETWEEN INTERSECTING LINES OR RAYS.

LINEAR PAIR - WHEN TWO ANGLES ARE DRAWN IN SUCH A WAY THAT THEY HELP TO FORM A LINE THEY FORM THIS SUPPLEMENTARY RELATIONSHIP.

SUPPLEMENTARY ANGLES - THE SUM OF THE ANGLES WILL BE 180°

EXAMPLE:



$$m\angle AFB + m\angle BFC = m\angle AFC \quad ; \quad m\angle AFC \text{ IS LINEAR} = 180^\circ$$

$$5x+5 + 3x-25 = 180$$

$$8x - 20 = 180$$

$$8x = 180 + 20$$

$$\frac{8x}{8} = \frac{200}{8}$$

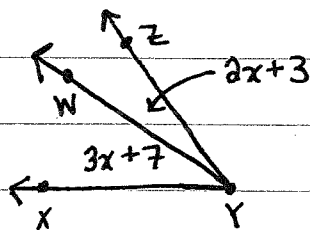
$$x = 25$$

$$\begin{aligned} m\angle AFB &= 5x+5 \\ &= 5(25)+5 \\ &= 130^\circ \end{aligned}$$

$$\begin{aligned} m\angle BFC &= 3x-25 \\ &= 3(25)-25 \\ &= 50^\circ \end{aligned}$$

ANGLE ADDITION

EXAMPLE $m\angle XYZ = 70$



$$m\angle XYW + m\angle WYZ = m\angle XYZ$$

$$3x+7 + 2x+3 = 70$$

$$5x + 10 = 70$$

$$5x = 70 - 10$$

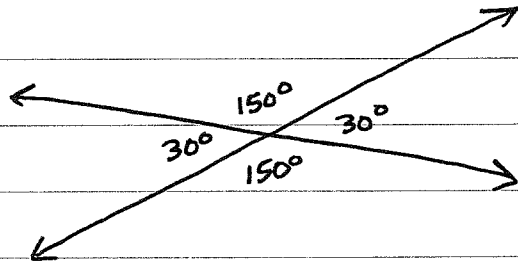
$$\frac{5x}{5} = \frac{60}{5}$$

$$x = 12$$

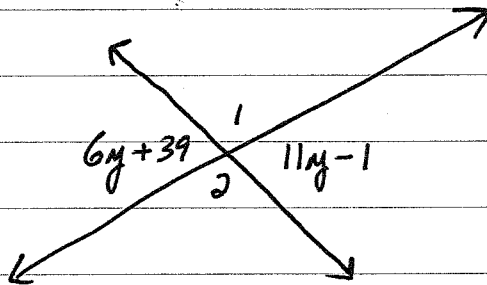
$$\begin{aligned} m\angle XYW &= 3x+7 & m\angle WYZ &= 2x+3 \\ &= 3(12)+7 & &= 2(12)+3 \\ &= 36+7 & &= 24+3 \\ &= 43^\circ & &= 27^\circ \end{aligned}$$

VERTICAL ANGLES - WHEN TWO LINES INTERSECT THE RESULT WILL BE TWO SETS OF ANGLES WITH EQUAL MEASURES. IN OTHER WORDS, THE NONSUPPLEMENTAL ANGLES WILL BE EQUAL.

EXAMPLE 1:



EXAMPLE 2:



$$6m + 39 = 11m - 1$$

$$39 + 1 = 11m - 6m$$

$$40 = 5m$$

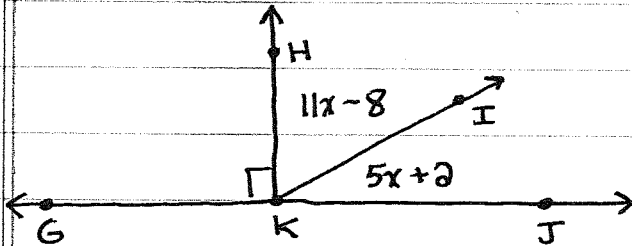
$$8 = m$$

$$\text{So } 11(8) - 1 = 87^\circ$$

$$\text{AND } m \angle 1 = 180 - 87 = 93^\circ$$

COMPLEMENTARY ANGLES - WHEN ANGLES ARE COMBINED TO FORM A RIGHT ANGLE, THE SUM WILL EQUAL 90°

EXAMPLE:



$$m\angle IKJ + m\angle HKI = m\angle HKJ$$

$$5x+2 + 11x-8 = 90$$

$$16x - 6 = 90$$

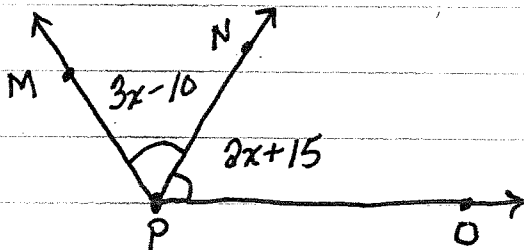
$$16x = 90+6$$

$$\begin{aligned} \text{So } m\angle IKJ &= 5x+2 \text{ AND } m\angle HKI \\ &= 5(6)+2 &= 90-32 \\ &= 32^\circ &= 58^\circ \end{aligned}$$

$$\begin{aligned} \frac{16x}{16} &= \frac{96}{16} \\ x &= 6 \end{aligned}$$

* INSTEAD OF PLUGGING $x=6$ BACK INTO EACH EXPRESSION YOU COULD FIND THE ANSWER FOR ONE OF THE EXPRESSIONS THEN USE THE COMPLEMENTARY PROPERTY TO FIND THE OTHER

ANGLE BISECTOR - MUCH AS A MIDPOINT DIVIDES A SEGMENT INTO TWO EQUAL PARTS, THE ANGLE BISECTOR WILL DIVIDE THE ANGLE INTO TWO ANGLES OF EQUAL MEASURE.



IF \vec{NP} BISECTS $\angle MPO$

$$\text{THEN } m\angle MPN = m\angle NPO$$

$$3x-10 = 2x+15$$

$$3x-2x = 15+10$$

$$x = 25$$

$$\begin{aligned} \text{So } m\angle MPN &= m\angle NPO = 2x+15 \\ &= 2(25)+15 \\ &= 65^\circ \end{aligned}$$

ADDITIONALLY ONE COULD FIGURE

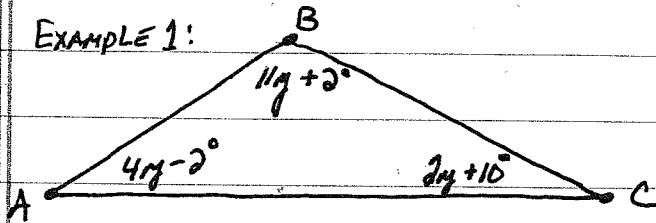
$$m\angle MPO = 130^\circ$$

TRIANGLES

ANGLE SUM FOR TRIANGLE - THE SUM OF THE ANGLES (INSIDE)

THE TRIANGLE MUST BE 180°

EXAMPLE 1:



$$m\angle A + m\angle B + m\angle C = 180$$

$$4n - 2 + 11n + 2 + 2n + 10 = 180$$

$$17n + 10 = 180$$

$$17n = 180 - 10$$

$$17n = 170$$

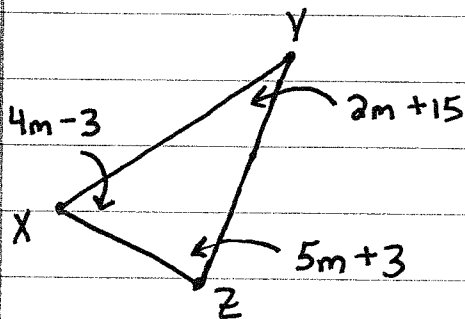
$$n = 10$$

$$m\angle A = 4n - 2 = 38^\circ$$

$$m\angle B = 11n + 2 = 112^\circ$$

$$m\angle C = 2n + 10 = 30^\circ$$

EXAMPLE 2



$$m\angle X + m\angle Y + m\angle Z = 180$$

$$4m - 3 + 2m + 15 + 5m + 3 = 180$$

$$11m + 15 = 180$$

$$11m = 180 - 15$$

$$11m = 165$$

$$m = 15$$

$$m\angle X = 4m - 3 = 4(15) - 3 = 57^\circ$$

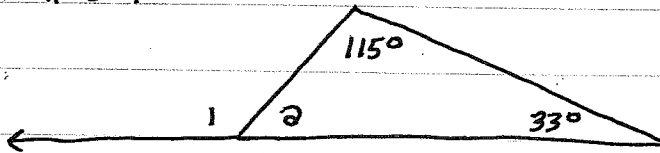
$$m\angle Y = 2m + 15 = 2(15) + 15 = 45^\circ$$

$$m\angle Z = 5m + 3 = 5(15) + 3 = 78^\circ$$

EXTERIOR ANGLE THEOREM

— THE ANGLE EXTERIOR AND ADJACENT TO AN INTERIOR ANGLE WILL FORM A SUPPLEMENTARY RELATIONSHIP, AND THE EXTERIOR ANGLE WILL BE EQUAL TO THE SUM OF THE TWO NON-ADJACENT INTERIOR ANGLES.

EXAMPLE 1



$$m\angle 1 + m\angle 2 = 180$$

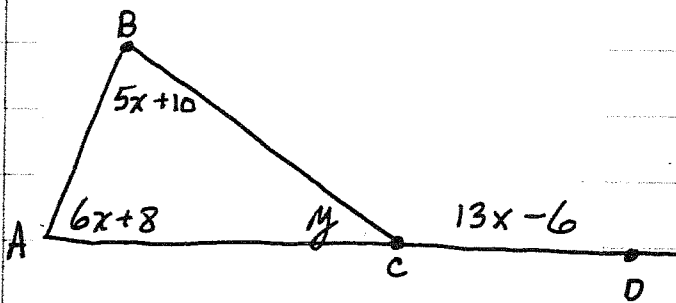
$$m\angle 1 = 180 - m\angle 2$$

$$m\angle 1 = 115 + 33$$

$$m\angle 1 = 148^\circ$$

$$\text{AND SO } m\angle 2 = 32^\circ$$

EXAMPLE 2



$$m\angle BCD = m\angle ABC + m\angle CAB$$

$$13x - 6 = 5x + 10 + 6x + 8$$

$$13x - 6 = 11x + 18$$

$$13x - 11x = 18 + 6$$

$$2x = 24$$

$$x = 12$$

$$m\angle ABC = 5x + 10 = 5(12) + 10 = 70^\circ$$

$$m\angle BAC = 6x + 8 = 6(12) + 8 = 80^\circ$$

$$m\angle BCD = 13x - 6 = 13(12) - 6 = 150^\circ$$

$$m\angle M = 30^\circ$$